TECHNICAL REPORT #5:

Developing Measures for Monitoring Progress in Elementary Grade Mathematics: An Investigation of Desirable Characteristics

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Developing Measures for Monitoring Progress in Elementary Grade Mathematics: An Investigation of Desirable Characteristics

Previous research on curriculum-based measurement in mathematics has focused on assessing grade-level math skills and knowledge sampled from the yearly curriculum. Most measures have included only grade-level computation skills (e.g., Shinn & Marston, 1985; Skiba, Magnusson, Marston, & Erickson, 1986; Fuchs, Hamlett, & Fuchs, 1990; Fuchs, Fuchs, Hamlett, Walz, & Germann, 1993; Thurber, Shinn, & Smolkowski, 2002; Hintze, Christ, & Keller, 2002; Evans-Hampton, Skinner, Henington, Sims, & McDaniel, 2002). A small number of measures have included grade-level concepts and applications (Helwig & Tindal, 2002; Helwig, Anderson, & Tindal, 2002; Fuchs, Hamlett, & Fuchs, 1994).

An alternate vein of progress monitoring measures for math has emerged, this not based upon systematic sampling of a grade level curriculum in math but instead on an effort to find a successful indicator of a generalized mathematics proficiency. Such "general outcome measures" have been investigated for younger students (preschool through first grade) based on tasks requiring basic "numeracy" (Chard, Clarke, Baker, Otterstedt, Braun, & Katz, 2004; Clarke & Shinn, 2004; VanDerHeyden, Broussard, Fabre, Stanley, LeGendre, & Creppell, 2004; VanDerHeyden, Witt, & Naquin, 2003; VanDerHeyden, Witt, Naquin, & Noell, 2001). With middle school students, general outcome measures involving estimation have been examined as well (Foegen, 2000; Foegen & Deno, 2001). With few exceptions (Espin, Deno, Maruyama, & Cohen, 1989; Thurber, Shinn, & Smolkowski, 2002), a general outcome measurement approach has not been applied at the elementary school level.

Given current standards and principles in mathematics education, criticism of an approach to measuring math proficiency via computational skill alone may be fitting. The National Council of Teachers of Mathematics (2000) emphasizes the importance in contemporary mathematical applications of problem solving, conceptual understandings, and computational fluency emphasizing mental math skills. The field of math education has increasingly focused on "number sense." Number sense, which Greeno (1991) defines as a "general condition of knowing in the domain of numbers and quantities," includes "flexible mental computation, numerical estimation, and quantitative judgement" (p. 170). It is possible that some students may develop strong computational skills without developing strength in number sense (Reys & Yang, 1998), but number sense is often understood as highly important to success in mathematical development, including computation. Gersten and Chard (1999) offer number sense as the "phonemic awareness" for mathematics, arguing that students who lack a strong base of flexible mental sense about numbers and quantities are those who may struggle more in mathematics.

Mental computation and computational fluency involve knowledge of basic facts and procedures, but the use of that knowledge today is understood to be different from that of the past, where figuring computation problems on paper was more relevant to real life mathematics (Woodward & Montague, 2002). Today, basic facts knowledge is used more in "mental computation and estimation" (p. 94). Fluency or automaticity with basic computational skills may support one's ability to succeed at higher level tasks such as problem-solving (Gagne, 1983; Hasselbring, Goin, & Bransford, 1988). Renewed engagement with this perspective within math reform efforts is evidenced by a recent (February 2003) special issue of NCTM's journal *Teaching Children Mathematics*, that

focus on teaching for computational fluency. It is not clear that sampling a student's grade-level computational curriculum is meant as a gauge of fluency; instead, the sampled items will necessarily involve types of computation taught later in the year, on which we would not expect students to demonstrate any degree of fluency before they are taught.

Although curriculum-based measures designed to sample concepts and applications may be more reflective of current understandings of mathematics proficiency, two important limitations to these curriculum-sampled measures exist. First, the practice of sampling each grade level curriculum does not lend itself to development of a measure appropriate for gauging cross-grade growth; instead, the measure that students are given in second grade, for example, will differ from that given in third grade. Because the measure and metric change, teachers and administrators cannot examine progress across, for example, all primary grades for a single student.

A second limitation, which applies most particularly to curriculum sampling of concepts and application, is that the duration of a sample is long relative to other curriculum-based measures. In the Fuchs, Hamlett, and Fuchs (1994) measures, for example, a single administration is eight minutes at second grade and seven at fifth grade. This length may limit the frequency with which they are administered.

Purpose of Study

The purpose of this study was to investigate the technical adequacy and utility of seven potential general outcome measures in mathematics. Specifically, we investigate the reliability, validity, sensitivity to growth within and across years, efficiency of administration time and ease of scoring. To gauge reliability of each measure, alternate-form and test-retest reliability of scores on single forms and of aggregated scores on two

and three forms was investigated. To gauge validity, the relation between the fall and spring CBM measures and district and state test scores and teacher ratings was investigated. To gauge sensitivity to growth, mean differences from fall to spring within each grade were examined and change in mean scores across grades were examined. Finally, time of administration was compared across measures, and interscorer reliability was investigated.

Method

Participants

This study took place in one elementary school and one middle school in an urban Midwestern district. Five classrooms from Grade 2, four classrooms from Grade 4, and five classrooms from Grade 6 participated in the study. Only scores from those offering consent were included in the study, making the sample size at each grade level as follows: Grade 2, n = 113; Grade 4, n = 96; and Grade 6, n = 89.

Demographics for the Grade 2 and 4 sample are given and compared to those for the whole school below:

	Grades 2 & 4 Sample	**Schoolwide**
Special education services	11%	8%
English Language Learner services	7%	9%
Free / reduced price lunch eligibility	18%	20%
Native American	1%	1%
African American	17%	18%
Asian	10%	9%
Hispanic	5%	5%
White	67%	66%
Female	51%	?

Demographics for the Grade 6 sample are given and compared to those for the whole school below:

	Grade 6 Sample	**Schoolwide**
Special education services	3%	17%
English Language Learner services	8%	11%
Free / reduced price lunch eligibility	42%	50%
Native American	3%	3%
African American	34%	40%
Asian	3%	5%
Hispanic	4%	5%
White	54%	47%
Female	60%	?

CBM Measures

Below we describe each of the CBM measures used in the study. A sample of each measure with administration instructions is provided in Appendix A.

Basic Facts. Problems on this measure included vertically formatted basic facts problems for all four computation operations. Only facts which can be generated by adding or multiplying numbers zero to nine were included; 9 + 9 = ? and 18 – 9 = ?, for example, are both within the field of items from which 20 problems each for addition, subtraction, multiplication, and division were drawn. Each form included 80 problems.

Cloze Math. Cloze items were drawn from the same field of basic math fact problems as described for the Basic Facts measure; however, these horizontally formatted problems left the blank for students to fill in at varying number locations within the math problems. 9 + ? = 18, ? – 9 = 9, and 18 ÷ 2 = ?, for example, are each within the field of problems from which 10 problems were randomly drawn for each operation. Order of problems was random; however, the location of the blank was rotated systematically so

that it did not occur in the same place for two consecutive problems. Each form included 40 problems.

Missing Number in Pattern. Each item on this measure represented a sequenced number pattern with three numbers present and the fourth missing for participants to fill in. Patterns were linear and increasing. They included multiples of numbers from one to ten (for example, "12, 18, 24, ?") and other numbers increasing by two or by ten (for example, "7, 9, 11, ?" and "32, 42, 52, ?"). Forty-four items were randomly selected from the field of all possible items for each form. The field of all possible sequenced number patterns is included in Appendix B.

Mixed Numeracy. Problems on this measure were sampled from Basic Facts, Cloze Math, and Missing Number items and compiled to form one new measure. Seventeen problems were randomly sampled from each type, with Cloze items sampled being limited to those with blanks in a location before the equal sign (those with blanks after the equal sign being essentially basic facts problems). Sampled items were placed randomly on the page, except that control was exerted to ensure that no more than two items of the same type appeared consecutively. Each form included 50 problems.

Quantity Arrays. Items on this measure were made up of dots arranged in arrays, with the task being for participants to write how many dots were present. Either one or two arrays were presented per item, with the total number of dots being up to 20. Each form included 42 items randomly selected from a field of all possible items.

Complex Quantity Discrimination. In this measure, two numeric expressions were arranged on either side of an empty box, and the direction was to fill the box with a greater than, less than, or equal sign as appropriate. Numeric expressions were either both single numbers, both simple operations, or one of each. A field of expression types was

generated for each operation (included in Appendix B), and stratified random sampling ensured that one third of items were single number to single number comparisons, one third were single number to simple operation comparisons, and the remaining third were comparisons between two like simple operations. For comparisons including operations, equal numbers of addition, subtraction, multiplication, and division types were represented. Each form included 40 comparison items.

Easy Estimation. This measure, a simplified version of an estimation task developed by Foegen and Deno (2001) for middle school students, included both strictly numeric and text-based problems. For each, the participants were instructed to select from three possible answers the choice representing a best estimate. Possible answers differed from each other by some factor of ten; choices might include, for example, "3, 30, or 300." Forty items were on each form.

Criterion Measures

Below we describe each of the criterion measures used in the study.

Teacher ratings. Given a list of their students and a one to seven point scale, teachers were asked to rate the overall math proficiency of their students relative to one another. Directions included a request that they use the full scale. The Teacher Rating of Students' Math Proficiency is included in Appendix C.

Northwest Achievement Levels Test (NALT) in Mathematics. All second through seventh graders who were considered capable of testing in the urban district where this study took place were administered an achievement-level version of the NALT Math, a multiple-choice achievement test. Problems included computation and number concepts (e.g. place value), geometry, and applications such as time and measurement.

Procedure

Progress monitoring measures. All progress monitoring probes were group administered during math class by researchers to participants at all three grade levels. Administration occurred on two days each week for two consecutive weeks in late fall (November/December for second and fourth grades, early December for sixth grade) and one week in spring (mid April for second and fourth grades, mid May for sixth grade). On each of the days, three forms of three types of measure were administered.

In fall, one week test-retest reliability was gauged for each of the measures except for Mixed Numeracy, which was developed based on fall findings to replace Quantity Arrays. The second week, each measure was re-administered exactly one week after the first administration. In spring, all progress monitoring measures except Quantity Arrays were administered for one week only. (Quantity Arrays was found in fall to have limited validity compared to the criterion of teacher ratings, and was therefore dropped for spring data collection.)

Order of measure type was counterbalanced across classrooms. Order of forms for each measure was counterbalanced across participants, with each participant taking forms in the same order as week one for week two administrations.

Apart from Easy Estimation, for which two minutes were allocated for each probe, all other measures were administered for one minute per probe. For each type of measure, an examples and practice page was presented to the group before the three timed forms were administered. Directions and the examples / practice page for each measure are included in Appendix D.

Criterion measures. Teacher ratings of students' math proficiency were collected in fall and again in spring.

The NALT was administered by school district personnel to all three grades in March. The MCA-4 was administered by the district in April; MCA-6 was administered by the district to sixth graders in May.

Results

Descriptive statistics

Descriptive statistics for all progress monitoring measures administered at grades 2, 4, and 6 are presented in Tables 1 through 7. Included are means, standard deviations, and sample sizes for number correct on each of three forms, average number correct on three forms, median correct on three forms, and average number incorrect on three forms. Administrations of three forms in each of Week 1 and Week 2 in fall and one week in spring are shown. Quantity Arrays was administered only in the two weeks in fall; Mixed Numeracy was administered only in spring.

Reliability

Alternate form. Alternate form reliability at each grade for each of the three separate weeks of administration is shown in Table 8, computed as Pearson's product-moment correlations. Sample sizes represent only participants for whom scores on all three forms were obtained. Note that findings for Quantity Arrays and Mixed Numeracy are limited to weeks administered.

Test-retest. One week test-retest reliability coefficients for each grade on each measure are presented in Table 9. Reliability coefficients for single form scores, average of scores on two forms, and average and median of scores on all three forms are included. Sample sizes represent only participants for whom scores on all three forms were obtained. Note that Mixed Numeracy is excluded because it was not administered until the single week in spring, and not retested.

Criterion validity

Achievement tests. Predictive and concurrent validity of the progress monitoring measures to the criteria of spring NALT and MCA tests are shown in Tables 10 and 11. Progress monitoring measure scores used in analysis are averages of number correct on each of three forms. Fall scores in Table 10 are taken from Week 1 and correlated to spring NALT and MCA scores to demonstrate predictive validity; spring scores in Table 11 are correlated to spring NALT and MCA scores to demonstrate concurrent validity. Note that predictive validity findings exclude Mixed Numeracy, not administered in fall, and that concurrent validity findings exclude Quantity Arrays, not administered in spring.

Teacher Ratings. Concurrent validity of progress measures to teacher ratings of mathematics proficiency is shown in Tables 12 and 13. Fall progress measures, which exclude Mixed Numeracy, are correlated to fall teacher ratings in Table 12. The progress measure scores used in analysis are average number correct on each of three forms administered in Week 1. Spring progress measures, which exclude Quantity Arrays, are correlated to spring teacher ratings in Table 13. Progress measure scores used are again an average number correct on each of three forms. Note that the results should be carefully interpreted at each grade level: At Grade 2, teachers rated a homeroom class of heterogeneous math proficiency. At Grade 4, teachers rated a math classroom grouped at one ability level. At Grade 6, students in five math classrooms (including classes at two levels) were combined and rated, by the teacher who taught all five classes, relative to the full group of students.

Growth

Across-grade growth. Because measures were constant across grade levels in the study, across-grade growth can be gauged by examining differences in means by grade in Tables 1 through 7.

Within-grade growth. Growth within the school year can be gauged for each measure by comparing fall and spring mean scores depicted in Tables 1 – 3, 6, and 7. Note that sixteen weeks of school occurred between the fall (week 1) and the spring administrations for students in second and fourth grades; nineteen weeks of school occurred between administrations for sixth grade students. Growth within each grade was analyzed by taking differences between mean scores from Week 1 in fall to mean scores in spring and standardizing these mean differences to obtain effect sizes. Effect sizes for captured growth on each measure are presented in Table 14.

Time

Administration time for each measure, as discussed in the Method section above, is presented for direct comparison in Table 15.

Scoring

Interscorer reliability (point by point, percentage agreements by item) was calculated for each measure on 5% of forms. These results are shown in Table 16.

Discussion

While the measures investigated in this study were selected as candidates for durability across elementary grades, some measures proved to work better across grades 2 through 6 than others. While Complex Quantity Discrimination (CQD) appears to be a measure with sound technical merits for students in intermediate grades, for instance, at second grade the scores produced by CQD were neither sufficiently reliable nor significantly correlated to validity criteria. With attention both to durability across grades

and also to use at particular grade levels, then, we discuss the reliability, validity, and growth reflected by each measure's scores.

Reliability

Reliability of scores on these measures was somewhat limited. For alternate-form reliability, all measures except Quantity Arrays (QA) produced coefficients above $r = .62$ for all weeks of administration. Missing Number (MN) was strongest at Grade 2; Cloze Math (CM) and Easy Estimation (EE) were strongest at Grade 4; and CM, MN, Mixed Numeracy (MX) and EE were all strong at Grade 6. One week test-retest reliability was improved by aggregation of scores from two or three forms; only after such aggregation were two measures at or above $r = .78$ for all grades, these measures being Basic Facts (BF) and MN. CM showed strong test-retest reliability at Grades 4 and 6, as well. It should be noted, however, that where individual decision-making might dictate a minimum reliability coefficient of $r = .90$, only at Grade 6 did measures prove adequate: BF, CM, and MN are all sufficiently reliable only after aggregation of scores from three forms. A possible contributor to the instability of scores on the developing measures investigated here may be the short administration time of just one minute per form (except EE, at 2 minutes).

Validity

Predictive and concurrent validity coefficients with the math achievement test as criterion were moderate or better across grades for three measures: MN, CM, and BF. In spring, when it was introduced, the MX measure also produced at least low moderate validity across grades. Other measures offered low moderate or moderate correlations at individual grade levels; CQD showed more promise as students were older, with low moderate correlations at Grade 4 and high moderate correlations at Grade 6. When

teacher ratings were the criterion, BF and CM were strongest across grades and MN and MX followed closely. Ratings at fourth grade generally correlated quite poorly to any of the measures: the highest correlations at Grade 4 are for CM, which had an average correlation to teacher ratings of $r = .37$ in fall and $r = .26$ in spring. These ratings were done within ability-grouped math classes, possibly making comparisons between students' math proficiency more challenging for teachers because students' proficiency was less variable. At sixth grade, however, all measures except QA produced at least moderate correlations to teacher ratings, with CQD again a high moderate correlation.

Growth

Across-grade growth appears to be gauged well by each of these measures, given the increase in means across grade levels sampled. For no measure was there a significant floor or ceiling effect preventing observable growth across these grades.

Within-grade growth was reflected differentially by the various measures. We note by examining the change in mean scores from late fall to early spring that students do not increase the number of items they answer correctly by much on CM or MN across the sixteen or nineteen weeks of this study, making the one minute sample appear to have low utility for monitoring progress. When growth across these weeks was standardized, CM apparently fails to capture growth commensurate with that reflected on the other measures for all grade levels, and EE fails to capture as much growth at Grade 6. The most growth is reflected by CQD; however, this presents an issue for further investigation since the validity of CM appears to be strong across grades while CQD is not sufficiently valid at Grade 2 and has weaker validity at Grade 4. Growth on a measure which is not valid is not growth of interest for monitoring progress in mathematics. Of the measures

valid across grades, the two which reflect growth competitively are MN and BF, with MN reflecting more growth.

Efficiency

Efficiency of administration and scoring for all measures investigated was strong, with all measures taking one or two minutes per form to administer and all exhibiting strong interscorer reliability.

Conclusions

Overall, the utility of MN, BF, and CM for all grades is indicated, although for progress monitoring CM appears from this study to have questionable use as a progress monitoring measure given the lower amount of growth it captured. Because scores on the CM measure are reliable and valid, however, further investigation into issues around growth is warranted. For MN, as well, slope of weekly growth appears to be low enough from scores on a one minute sample that further study, including changes in administration time, may be warranted. At sixth grade, CQD provides sufficient reliability, moderately strong validity, and strong reflection of growth, indicating utility as a progress measure at this grade level.

An important note applicable to all of the developing measures investigated is that the technical adequacy of these measures falls short of that found in other sample populations for some existing measures of math progress, particularly the Monitoring Basic Skills Progress math measures (Fuchs, Hamlett, & Fuchs, 1990; Fuchs, Hamlett, & Fuchs, 1994). While advantages remain in administration efficiency and in utility for gauging across-grade growth, none of these measures in this study surpassed the technical soundness of the MBSP measures for within-grade growth.

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Table 1
Basic Facts: Descriptive Statistics

	Fall, Week 1			Fall, Week 2			Spring		
	M	*SD*	*n*	***M***	*SD*	*n*	***M***	*SD*	*n*
Grade 2									
Number Correct:									
Form A	**6.94**	3.06	108	**7.31**	3.23	107	**8.22**	3.61	110
Form B	**6.56**	2.83	108	**7.06**	3.04	107	**7.84**	3.64	109
Form C	**5.54**	3.25	108	**6.58**	3.38	107	**7.55**	3.86	109
Average of 3	**6.35**	2.72	108	**6.96**	2.88	107	**7.84**	3.42	110
Median of 3	**6.42**	2.76	108	**6.93**	3.06	107	**7.76**	3.42	110
Number Incorrect:									
Average of 3	**1.08**	0.99	108	**1.06**	1.09	107	**1.01**	1.27	110
Grade 4									
Number Correct:									
Form A	**14.39**	4.66	87	**14.18**	4.89	90	**16.26**	6.06	91
Form B	**13.69**	4.93	87	**13.26**	5.09	90	**15.80**	5.67	91
Form C	**14.22**	5.89	87	**14.69**	5.86	90	**16.65**	6.83	91
Average of 3	**14.10**	4.64	87	**14.04**	4.90	90	**16.24**	5.82	91
Median of 3	**14.05**	4.75	87	**14.07**	5.24	90	**16.35**	5.83	91
Number Incorrect:									
Average of 3	**0.97**	0.84	87	**1.04**	1.49	90	**1.06**	1.51	91
Grade 6									
Number Correct:									
Form A	**20.77**	7.29	70	**22.52**	7.51	75	**24.14**	8.94	77
Form B	**21.09**	8.41	70	**22.92**	9.61	75	**23.88**	9.56	77
Form C	**23.04**	8.53	69	**23.95**	9.93	75	**26.29**	9.75	77
Average of 3	**21.57**	7.65	70	**23.13**	8.38	75	**24.77**	8.96	77
Median of 3	**21.44**	7.74	70	**23.27**	8.27	75	**24.71**	9.21	77
Number Incorrect:									
Average of 3	**1.18**	1.12	70	**1.06**	1.10	75	**1.11**	1.34	77

Table 2
Cloze Math: Descriptive Statistics

	Fall, Week 1			Fall, Week 2			Spring		
	M	*SD*	*n*	***M***	*SD*	*n*	***M***	*SD*	*n*
Grade 2									
Number Correct:									
Form A	**3.24**	2.32	105	**3.66**	2.30	107	**4.01**	2.75	108
Form B	**3.05**	2.80	105	**3.48**	2.72	107	**4.12**	3.09	108
Form C	**2.73**	2.41	105	**3.02**	2.41	106	**3.19**	2.90	109
Average of 3	**3.01**	2.23	105	**3.39**	2.26	107	**3.76**	2.63	109
Median of 3	**2.95**	2.38	105	**3.28**	2.28	107	**3.75**	2.79	109
Number Incorrect:									
Average of 3	**1.01**	0.74	105	**0.89**	0.96	107	**1.07**	1.52	109
Grade 4									
Number Correct:									
Form A	**8.88**	4.60	90	**9.66**	4.47	89	**10.30**	4.48	88
Form B	**8.81**	4.29	90	**9.31**	5.03	90	**10.02**	4.71	88
Form C	**8.01**	4.50	90	**8.09**	4.32	89	**8.83**	4.58	88
Average of 3	**8.57**	4.16	90	**8.99**	4.23	90	**9.72**	4.22	88
Median of 3	**8.56**	4.30	90	**8.97**	4.36	90	**9.92**	4.52	88
Number Incorrect:									
Average of 3	**1.10**	2.25	90	**0.99**	0.92	90	**0.99**	0.84	88
Grade 6									
Number Correct:									
Form A	**14.89**	6.35	75	**14.91**	6.44	79	**16.10**	7.23	79
Form B	**15.14**	6.42	76	**15.19**	7.50	79	**16.30**	7.33	80
Form C	**13.57**	7.27	76	**13.97**	7.10	79	**14.89**	7.93	79
Average of 3	**14.52**	6.36	76	**14.69**	6.70	79	**15.74**	7.09	80
Median of 3	**14.58**	6.38	76	**14.72**	6.70	79	**15.52**	7.22	80
Number Incorrect:									
Average of 3	**1.02**	0.91	76	**1.00**	0.92	79	**1.35**	3.42	79

Table 3
Missing Number in Pattern: Descriptive Statistics

	Fall, Week 1			Fall, Week 2			Spring		
	M	*SD*	*n*	***M***	*SD*	*n*	***M***	*SD*	*n*
Grade 2									
Number Correct:									
Form A	**5.98**	3.77	105	**6.82**	3.72	101	**7.79**	3.87	107
Form B	**6.51**	4.07	111	**7.26**	4.24	107	**8.58**	4.23	107
Form C	**7.64**	3.99	101	**8.47**	3.96	98	**9.18**	3.53	107
Average of 3	**6.62**	3.62	111	**7.37**	3.74	108	**8.52**	3.55	107
Median of 3	**6.61**	3.65	111	**7.36**	3.89	108	**8.52**	3.73	107
Number Incorrect:									
Average of 3	**0.60**	1.13	111	**0.50**	0.78	108	**0.53**	1.14	107
Grade 4									
Number Correct:									
Form A	**10.14**	3.79	79	**11.24**	4.20	82	**11.53**	4.50	89
Form B	**11.30**	4.75	87	**12.07**	4.57	90	**12.58**	4.82	89
Form C	**11.18**	4.50	79	**12.44**	4.60	81	**13.45**	4.46	88
Average of 3	**11.06**	4.02	87	**12.08**	4.14	90	**12.49**	4.24	89
Median of 3	**11.05**	4.24	87	**12.05**	4.27	90	**12.61**	4.61	89
Number Incorrect:									
Average of 3	**0.53**	0.67	87	**0.51**	0.56	90	**0.61**	1.56	89
Grade 6									
Number Correct:									
Form A	**15.61**	6.33	69	**16.46**	7.42	69	**17.98**	6.69	80
Form B	**16.97**	5.43	72	**17.69**	5.76	72	**18.68**	6.01	79
Form C	**18.37**	5.84	68	**19.15**	6.22	71	**19.58**	6.89	81
Average of 3	**16.87**	5.51	72	**17.70**	6.17	72	**18.56**	6.47	81
Median of 3	**16.88**	5.50	72	**17.90**	6.23	72	**18.57**	6.52	81
Number Incorrect:									
Average of 3	**0.92**	2.61	72	**1.42**	4.64	72	**0.94**	2.39	81

Table 4
Quantity Arrays: Descriptive Statistics

	Fall, Week 1			Fall, Week 2		
	M	*SD*	*n*	***M***	*SD*	*n*
Grade 2						
Number Correct:						
Form A	**11.04**	3.79	80	**11.21**	3.58	76
Form B	**11.50**	3.75	80	**11.64**	3.69	77
Form C	**12.41**	3.04	80	**13.05**	2.80	76
Average of 3	**11.65**	3.16	80	**11.91**	3.03	77
Median of 3	**11.63**	3.34	80	**12.17**	3.05	77
Number Incorrect:						
Average of 3	**0.63**	0.84	80	**0.71**	0.80	76
Grade 4						
Number Correct:						
Form A	**13.74**	3.95	74	**14.91**	3.18	77
Form B	**14.62**	3.06	73	**15.13**	2.79	77
Form C	**14.92**	3.55	74	**16.09**	2.65	77
Average of 3	**14.45**	2.64	74	**15.38**	2.51	77
Median of 3	**14.59**	2.65	74	**15.47**	2.43	77
Number Incorrect:						
Average of 3	**0.44**	0.47	74	**0.51**	0.67	77
Grade 6						
Number Correct:						
Form A	**18.88**	6.06	48	**17.98**	5.33	53
Form B	**17.92**	4.42	49	**19.00**	4.75	53
Form C	**20.67**	5.80	48	**19.36**	4.48	53
Average of 3	**19.05**	4.17	49	**18.78**	4.48	53
Median of 3	**18.57**	4.37	49	**18.77**	4.43	53
Number Incorrect:						
Average of 3	**0.84**	1.04	49	**0.82**	0.92	53

Table 5
Mixed Numeracy: Descriptive Statistics

	Spring		
	M	*SD*	*n*
Grade 2			
Number Correct:			
Form A	**5.60**	2.94	111
Form B	**6.66**	3.06	111
Form C	**8.55**	2.81	111
Average of 3	**6.94**	2.65	111
Median of 3	**6.92**	2.94	111
Number Incorrect:			
Average of 3	**0.71**	1.45	111
Grade 4			
Number Correct:			
Form A	**12.94**	7.62	88
Form B	**13.72**	7.85	88
Form C	**14.48**	5.80	88
Average of 3	**13.71**	5.85	88
Median of 3	**12.95**	5.10	88
Number Incorrect:			
Average of 3	**1.00**	0.94	88
Grade 6			
Number Correct:			
Form A	**19.83**	8.20	78
Form B	**19.85**	7.71	78
Form C	**21.92**	7.52	78
Average of 3	**20.53**	7.42	78
Median of 3	**20.40**	7.84	78
Number Incorrect:			
Average of 3	**1.10**	1.02	78

Table 6
Easy Estimation: Descriptive Statistics

	Fall, Week 1			Fall, Week 2			Spring		
	M	*SD*	*n*	***M***	*SD*	*n*	***M***	*SD*	*n*
Grade 2									
Number Correct:									
Form A	**7.44**	3.78	109	**8.58**	4.13	107	**9.57**	4.19	108
Form B	**6.17**	3.61	108	**6.92**	4.20	102	**8.17**	4.34	109
Form C	**7.00**	3.10	108	**7.80**	3.39	101	**8.65**	3.47	108
Average of 3	**6.87**	3.06	109	**7.89**	3.70	108	**8.76**	3.65	109
Median of 3	**6.91**	3.17	109	**7.81**	3.73	108	**8.80**	3.75	109
Number Incorrect:									
Average of 3	**3.22**	5.57	109	**3.01**	4.79	108	**2.71**	3.44	109
Grade 4									
Number Correct:									
Form A	**12.01**	5.22	88	**14.60**	6.35	90	**15.94**	6.22	87
Form B	**10.05**	4.82	74	**12.84**	5.62	77	**14.00**	5.38	87
Form C	**10.77**	4.27	74	**13.77**	5.14	77	**15.37**	5.25	87
Average of 3	**11.19**	4.44	88	**13.94**	5.41	91	**15.10**	5.21	87
Median of 3	**11.28**	4.52	88	**13.93**	5.37	91	**14.91**	5.17	87
Number Incorrect:									
Average of 3	**2.93**	3.58	88	**4.25**	5.01	91	**4.52**	4.63	87
Grade 6									
Number Correct:									
Form A	**20.05**	7.18	76	**21.29**	6.60	75	**22.05**	7.61	82
Form B	**15.69**	7.97	48	**19.22**	7.60	49	**18.84**	7.83	82
Form C	**16.22**	7.08	49	**18.88**	6.71	48	**18.96**	6.06	82
Average of 3	**18.75**	7.09	76	**20.26**	6.52	76	**19.95**	6.75	82
Median of 3	**18.64**	7.12	76	**20.42**	6.43	76	**19.94**	7.09	82
Number Incorrect:									
Average of 3	**3.04**	3.79	76	**3.52**	4.54	76	**3.63**	3.43	82

Table 7
Complex Quantity Discrimination: Descriptive Statistics

	Fall, Week 1			Fall, Week 2			Spring		
	M	*SD*	*n*	***M***	*SD*	*n*	***M***	*SD*	*n*
Grade 2									
Number Correct:									
Form A	**6.77**	3.51	111	**8.49**	3.62	105	**9.70**	4.45	110
Form B	**7.27**	3.17	111	**8.38**	4.19	105	**9.64**	4.09	110
Form C	**5.96**	3.50	111	**7.74**	4.27	106	**8.04**	4.54	110
Average of 3	**6.67**	2.98	111	**8.17**	3.64	106	**9.12**	3.93	110
Median of 3	**6.68**	3.21	111	**8.18**	3.69	106	**9.14**	3.81	110
Number Incorrect:									
Average of 3	**1.72**	1.95	111	**1.83**	2.99	106	**1.48**	1.96	110
Grade 4									
Number Correct:									
Form A	**12.79**	5.03	85	**14.91**	5.04	89	**16.93**	5.64	89
Form B	**11.94**	4.20	85	**14.36**	4.60	89	**15.48**	5.74	87
Form C	**11.65**	5.15	85	**14.78**	5.84	90	**15.92**	6.26	89
Average of 3	**12.13**	4.49	85	**14.60**	4.76	90	**16.17**	5.55	89
Median of 3	**12.12**	4.51	85	**14.74**	4.94	90	**16.30**	5.70	89
Number Incorrect:									
Average of 3	**1.43**	2.26	85	**1.68**	3.04	90	**1.95**	4.01	89
Grade 6									
Number Correct:									
Form A	**19.26**	5.25	72	**21.64**	5.18	73	**21.95**	6.45	79
Form B	**17.24**	5.08	72	**20.10**	5.68	73	**22.08**	6.95	79
Form C	**18.48**	5.74	71	**21.29**	5.26	73	**21.90**	7.13	79
Average of 3	**18.32**	4.79	72	**21.01**	4.96	73	**21.97**	6.37	79
Median of 3	**18.42**	4.89	72	**21.00**	5.18	73	**22.11**	6.26	79
Number Incorrect:									
Average of 3	**1.66**	3.48	72	**1.36**	3.28	73	**1.54**	3.51	79

Table 8
Alternate Form Reliability

	Fall, Week 1		Fall, Week 2		Spring	
	r	*n*	***r***	*n*	***r***	*n*
Basic Facts						
Grade 2	**.72, .68, .68**	108	**.75, .65, .65**	106	**.79, .77, .74**	109
Grade 4	**.71, .65, .78**	87	**.76, .83, .78**	90	**.82, .82, .84**	91
Grade 6	**.82, .85, .84**	69	**.76, .84, .78**	75	**.85, .87, .86**	77
Cloze Math						
Grade 2	**.64, .64, .77**	105	**.72, .78, .76**	106	**.71, .71, .75**	107
Grade 4	**.83, .79, .79**	90	**.74, .79, .75**	89	**.79, .77, .75**	88
Grade 6	**.86, .85, .88**	75	**.86, .86, .88**	79	**.83, .91, .81**	79
Missing Number						
Grade 2	**.77, .75, .75**	95	**.80, .81, .80**	91	**.74, .81, .74**	107
Grade 4	**.63, .69, .80**	71	**.80, .71, .77**	73	**.75, .73, .79**	88
Grade 6	**.85, .85, .83**	65	**.84, .88, .89**	68	**.88, .89, .89**	79
Quantity Arrays						
Grade 2	**.70, .73, .68**	80	**.63, .69, .74**	76		
Grade 4	**.45, .31, .20**	73	**.56, .68, .71**	77		
Grade 6	**.47, .18, .45**	48	**.76, .79, .79**	53		
Mixed Numeracy						
Grade 2					**.77, .66, .72**	111
Grade 4					**.52, .46, .58**	88
Grade 6					**.90, .83, .83**	78
Easy Estimation						
Grade 2	**.69, .64, .62**	108	**.70, .66, .70**	101	**.74, .72, .73**	108
Grade 4	**.76, .81, .71**	74	**.76, .81, .81**	76	**.78, .80, .78**	87
Grade 6	**.83, .82, .85**	48	**.89, .88, .80**	48	**.80, .84, .86**	82
Complex Quantity Discrim.						
Grade 2	**.69, .66, .62**	111	**.76, .75, .65**	105	**.76, .68, .72**	110
Grade 4	**.80, .84, .80**	85	**.68, .82, .68**	89	**.78, .82, .81**	87
Grade 6	**.63, .76, .73**	71	**.78, .79, .77**	73	**.75, .81, .84**	79

Table 9
Test-Retest Reliability

	Single Forms		Average of 2 forms		Average of 3 forms		Median of 3 forms	
	r	*n*	*r*	*n*	*r*	*n*	*r*	*n*
Basic Facts								
Grade 2	.74, .76, .69	103	.82, .80, .79	103	.83	103	.78	103
Grade 4	.66, .72, .71	85	.77, .76, .77	85	.79	85	.79	85
Grade 6	.86, .79, .86	68	.88, .91, .89	68	.91	68	.88	68
Cloze Math								
Grade 2	.38, .69, .61	100	.64, .58, .73	101	.69	101	.60	101
Grade 4	.80, .77, .76	87	.84, .86, .83	87	.87	88	.87	88
Grade 6	.83, .84, .85	75	.88, .89, .88	75	.90	75	.86	75
Missing Number								
Grade 2	.81, .80, .78	96	.86, .85, .86	106	.89	107	.87	107
Grade 4	.61, .73, .80	76	.75, .75, .76	85	.78	85	.76	85
Grade 6	.85, .85, .89	64	.89, .92, .92	67	.93	67	.91	67
Quantity Arrays								
Grade 2	.55, .52, .50	74	.61, .60, .58	74	.63	74	.59	74
Grade 4	.55, .40, .49	73	.60, .59, .66	73	.69	73	.64	73
Grade 6	.56, .71, .44	48	.73, .68, .70	48	.77	49	.85	49
Easy Estimation								
Grade 2	.55, .59, .55	99	.68, .67, .67	105	.72	105	.72	105
Grade 4	.72, .78, .74	73	.78, .77, .84	87	.80	87	.80	87
Grade 6	.75, .72, .75	71	.75, .76, .78	71	.76	72	.75	72
Complex Q'ty Discri								
Grade 2	.71, .56, .62	104	.69, .73, .68	104	.72	105	.70	105
Grade 4	.70, .72, .75	83	.77, .80, .83	83	.83	83	.80	83
Grade 6	.77, .65, .73	68	.77, .82, .76	68	.81	68	.78	68

Table 10
Predictive Validity of Fall Progress Measures to Spring Achievement Test

	NALT	
	r	*n*
Basic Facts		
Grade 2	**.53****	108
Grade 4	**.43****	85
Grade 6	**.58****	65
Cloze Math		
Grade 2	**.51****	105
Grade 4	**.59****	88
Grade 6	**.64****	72
Missing Number		
Grade 2	**.55****	111
Grade 4	**.59****	85
Grade 6	**.69****	68
Quantity Arrays		
Grade 2	**.40****	80
Grade 4	**.09**	73
Grade 6	**.33***	45
Easy Estimation		
Grade 2	**.09**	109
Grade 4	**.35****	86
Grade 6	**.59****	73
Complex Q'ty Discrim.		
Grade 2	**.31****	111
Grade 4	**.43****	83
Grade 6	**.62****	68

* significant, $p<.05$
** significant, $p<.01$
Note: Average of three progress measure probe scores correlated with standard scores from NALT.

Table 11
Concurrent Validity of Spring Progress Measures with Spring Achievement Test

	NALT	
	r	*n*
Basic Facts		
Grade 2	**.55****	110
Grade 4	**.53****	91
Grade 6	**.60****	73
Cloze Math		
Grade 2	**.54****	109
Grade 4	**.57****	88
Grade 6	**.68****	75
Missing Number		
Grade 2	**.49****	107
Grade 4	**.61****	89
Grade 6	**.66****	77
Mixed Numeracy		
Grade 2	**.40****	111
Grade 4	**.58****	88
Grade 6	**.66****	74
Easy Estimation		
Grade 2	**.40****	109
Grade 4	**.23***	87
Grade 6	**.50****	78
Complex Q'ty Discrim.		
Grade 2	**.37****	110
Grade 4	**.43****	89
Grade 6	**.66****	75

* significant, $p<.05$
** significant, $p<.01$
Note: Average of three progress measure probe scores correlated with standard scores from NALT.

Table 12
Concurrent Validity of Fall Progress Measures to Fall Teacher Ratings

	Teacher 1		Teacher 2		Teacher 3		Teacher 4		Teacher 5	
	r	*n*	***r***	*n*	***r***	*n*	***r***	*n*	***r***	*n*
Basic Facts										
Grade 2	**.37**	24	**.55***	20	**.59****	23	**.80****	20	**.57****	21
Grade 4	**.08**	27	**.46**	17	**.16**	18	**.24**	25		
Grade 6	**.46****	68								
Cloze Math										
Grade 2	**.44***	25	**.55***	20	**.47***	23	**.74****	17	**.47***	20
Grade 4	**.46***	27	**.09**	17	**.39**	21	**.55****	25		
Grade 6	**.47****	75								
Missing Number										
Grade 2	**.58****	26	**.64****	20	**.71****	24	**.60****	20	**.45***	21
Grade 4	**.13**	27	**.42**	17	**-.21**	18	**.30**	25		
Grade 6	**.52****	68								
Quantity Arrays										
Grade 2	**.45***	25	**.39**	21	**.36**	23	**.53**	5	**-.13**	6
Grade 4	**-.18**	25	**-.11**	17	**-.13**	7	**.51****	25		
Grade 6	**.22**	48								
Easy Estimation										
Grade 2	**.33**	25	**.33**	21	**-.21**	24	**.11**	18	**.08**	21
Grade 4	**.02**	25	**-.30**	17	**-.30**	21	**.21**	25		
Grade 6	**.50****	73								
Complex Quantity Discrimination										
Grade 2	**.37**	25	**.50***	21	**.29**	24	**.56****	20	**.18**	21
Grade 4	**.39**	25	**-.18**	17	**-.41**	18	**.30**	25		
Grade 6	**.49****	68								

* significant, $p<.05$
** significant, $p<.01$
Note: Average of three progress measure probe scores correlated with teacher ratings of student math proficiency. Grade 2 teacher ratings were within single homerooms, not grouped by math ability. Grade 4 were within single ability-leveled math classes. Grade 6 were rated as one group across five classrooms, same teacher.

Table 13
Concurrent Validity of Spring Progress Measures to Spring Teacher Ratings

Concurrent Validity, Spring Teacher Ratings	Teacher 1		Teacher 2		Teacher 3		Teacher 4		Teacher 5	
	r	*n*	***r***	*n*	***r***	*n*	***r***	*n*	***r***	*n*
Basic Facts										
Grade 2	**.22**	24	**.58****	21	**.78****	24	**.72****	20	**.59****	20
Grade 4	**.14**	29	**-.13**	14	**.41**	19	**.36**	26		
Grade 6	**.54****	75								
Cloze Math										
Grade 2	**.26**	24	**.66****	20	**.73****	24	**.69****	20	**.61****	20
Grade 4	**.21**	29	**-.23**	13	**.42**	17	**.63****	26		
Grade 6	**.63****	77								
Missing Number										
Grade 2	**.32**	23	**.56***	20	**.82****	24	**.56****	20	**.16**	20
Grade 4	**.15**	27	**-.22**	14	**.12**	19	**.51****	26		
Grade 6	**.57****	79								
Mixed Numeracy										
Grade 2	**.11**	24	**.60****	21	**.72****	24	**.59****	20	**.51***	21
Grade 4	**.00**	29	**-.03**	14	**.26**	16	**.49****	26		
Grade 6	**.58****	76								
Easy Estimation										
Grade 2	**.35**	23	**.53***	20	**.34**	24	**.64****	20	**.46***	21
Grade 4	**.07**	27	**.01**	14	**.01**	17	**.31**	26		
Grade 6	**.45****	80								
Complex Quantity Discrimination										
Grade 2	**.04**	23	**.68****	21	**.53****	24	**.68****	20	**.14**	21
Grade 4	**.31**	27	**-.25**	14	**-.20**	19	**.29**	26		
Grade 6	**.67****	77								

* significant, $p<.05$
** significant, $p<.01$
Note: Average of three progress measure probe scores correlated with teacher ratings of student math proficiency. Grade 2 teacher ratings were within single homerooms, not grouped by math ability. Grade 4 were within single ability-leveled math classes. Grade 6 were rated as one group across five classrooms, same teacher.

Table 14
Within Grade Growth on Each Measure

	Grade 2		Grade 4		Grade 6	
	Effect size	*n*	**Effect size**	*n*	**Effect size**	*n*
Basic Facts	**.56**	105	**.58**	82	**.69**	59
Cloze Math	**.40**	101	**.42**	82	**.42**	68
Missing Number	**.79**	105	**.53**	80	**.79**	65
Easy Estimation	**.56**	105	**.89**	80	**.38**	72
Complex Quantity Discrim.	**.87**	108	**1.35**	79	**.90**	63

Note: Effect sizes are standardized mean differences in number correct from average of three fall probes to average of three spring probes.
Note: Sixteen weeks of school occurred between administrations for grades 2 and 4; nineteen weeks of school occurred between administrations for grade 6.

Table 15
Administration Time for Single Form of Each Measure

Measure	Administration Time
Basic Facts	1 minute
Cloze Math	1 minute
Missing Number	1 minute
Quantity Arrays	1 minute
Mixed Numeracy	1 minute
Easy Estimation	2 minutes
Complex Quantity Discrimination	1 minute

Table 16
Interscorer Reliability for Each Measure

Measure	Reliability
Basic Facts	97.00
Cloze Math	96.79
Missing Number	99.49
Quantity Arrays	99.61
Mixed Numeracy	96.73
Easy Estimation	99.05
Complex Quantity Discrimination	97.33

Note: Based on 5% of probes administered. Calculation is percentage agreement for items within probes (point by point).

Appendix A: Form A of Each Measure

Basic Facts

Cloze Math

Missing Number

Mixed Numeracy

Quantity Arrays

Complex Quantity Discrimination

Easy Estimation

BF-A

1 × 7	5 - 0	5 + 2	14 - 8	4 + 7	1 × 3	4 - 2	$3\overline{)3}$	4 + 1	$7\overline{)14}$
0 + 2	$5\overline{)35}$	13 - 9	3 × 3	16 - 9	$4\overline{)20}$	9 × 6	5 + 5	$5\overline{)25}$	$9\overline{)72}$
7 × 5	7 × 4	15 - 7	7 + 1	7 × 3	$6\overline{)12}$	2 + 7	$6\overline{)54}$	6 - 1	8 × 6
5 + 9	11 - 9	0 × 2	$1\overline{)5}$	14 - 5	$4\overline{)32}$	6 × 4	8 - 5	6 + 6	1 + 9
6 × 9	8 - 4	2 × 8	$5\overline{)30}$	2 + 4	14 - 9	$5\overline{)20}$	1 + 5	$4\overline{)16}$	$3\overline{)6}$
1 + 0	9 - 3	10 - 5	9 + 8	7 × 6	6 × 2	0 + 7	$1\overline{)6}$	$4\overline{)8}$	2 × 7
12 - 4	11 - 8	$8\overline{)72}$	3 + 6	$7\overline{)56}$	6 × 7	$9\overline{)45}$	10 - 4	5 + 0	0 + 6
6 - 2	6 - 0	6 × 0	0 × 3	10 - 3	0 × 7	$8\overline{)32}$	6 + 3	3 + 8	7 × 4

CM-A

□ × 6 = 12	45 ÷ □ = 5	8 + 6 = □	□ × 9 = 0
3 × □ = 18	16 ÷ 4 = □	□ ÷ 3 = 9	64 ÷ □ = 8
0 + 1 = □	□ + 5 = 7	4 × □ = 8	3 – 1 = □
□ + 7 = 16	4 × □ = 36	0 ÷ 6 = □	□ – 9 = 5
9 × □ = 9	1 – 0 = □	□ + 7 = 14	6 + □ = 13
0 ÷ 8 = □	□ × 5 = 45	2 + □ = 5	1 × 4 = □
□ – 6 = 1	6 + □ = 8	8 ÷ 8 = □	□ – 9 = 0
8 × □ = 32	6 ÷ 6 = □	□ + 9 = 11	15 – □ = 7
1 + 0 = □	□ – 4 = 3	10 ÷ □ = 5	13 – 9 = □
□ – 0 = 8	42 ÷ □ = 6	7 × 9 = □	□ – 9 = 7

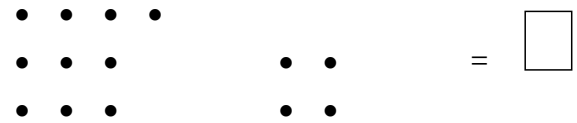
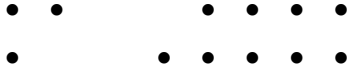
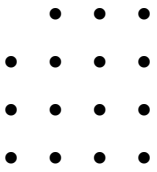
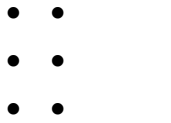
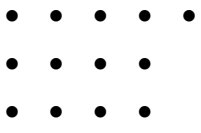
MN-A

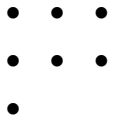
9, 12, 15, ___	9, 18, 27, ___	10, 20, 30, ___	4, 5, 6, ___
70, 80, 90, ___	10, 11, 12, ___	16, 24, 32, ___	9, 10, 11, ___
8, 16, 24, ___	9, 11, 13, ___	8, 10, 12, ___	13, 23, 33, ___
1, 2, 3, ___	11, 13, 15, ___	7, 8, 9, ___	29, 39, 49, ___
12, 14, 16, ___	7, 14, 21, ___	24, 30, 36, ___	11, 21, 31, ___
13, 14, 15, ___	14, 16, 18, ___	4, 6, 8, ___	18, 24, 30, ___
24, 32, 40, ___	30, 40, 50, ___	25, 30, 35, ___	6, 12, 18, ___
23, 33, 43, ___	5, 7, 9, ___	9, 19, 29, ___	6, 9, 12, ___
3, 4, 5, ___	12, 13, 14, ___	14, 15, 16, ___	2, 3, 4, ___
35, 40, 45, ___	10, 12, 14, ___	17, 18, 19, ___	3, 5, 7, ___
18, 27, 36, ___	17, 27, 37, ___	8, 12, 16, ___	6, 8, 10, ___

Mixed-A

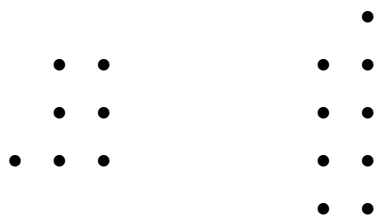
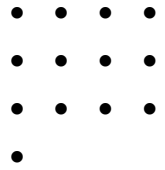
1 - 1	6, 12, 18, __	8 ÷ □ = 2	18, 27, 36, __	0 × 5
45 ÷ □ = 5	30, 40, 50, __	9 + 8	15, 25, 35, __	□ + 4 = 4
5, 15, 25, __	14 - 9	$4\overline{)0}$	1× □ = 6	12, 14, 16, __
13, 14, 15, __	5 + 5	9 + □ = 17	50, 60, 70, __	7 + 1
9 × 7	9 + □ = 10	9, 11, 13, __	3 × 1	2 × 4
□ ÷ 8 = 4	□ + 5 = 7	23, 33, 43, __	14, 15, 16, __	□ + 2 = 3
19, 29, 39, __	$7\overline{)56}$	3 × □ = 9	6, 9, 12, __	11 − □ = 4
□ − 8 = 5	5 - 4	□ × 6 = 48	□ + 1 = 8	$8\overline{)72}$
1, 11, 21, __	$1\overline{)4}$	70, 80, 90, __	□ × 9 = 0	7 × 8
16, 24, 32, __	0 × 7	□ − 6 = 1	8 + □ = 14	2, 4, 6, __

QA-A1

 = ☐	 = ☐

QA-A2

 = ☐	 = ☐
 = ☐	= ☐
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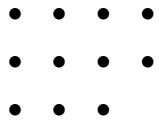
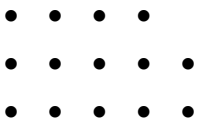
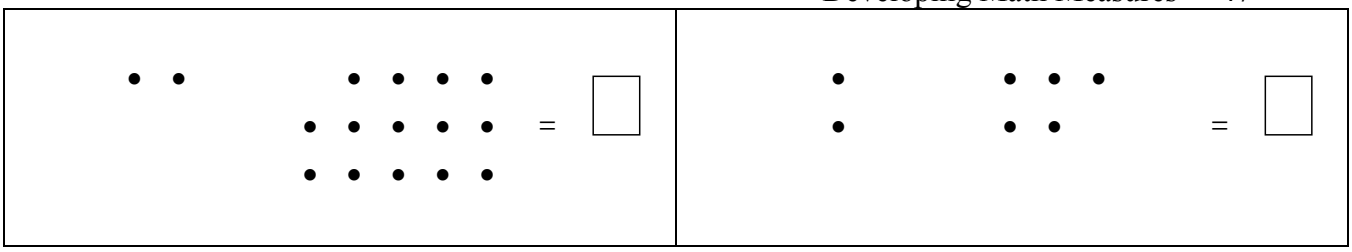
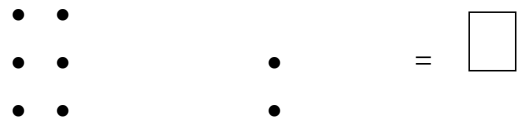
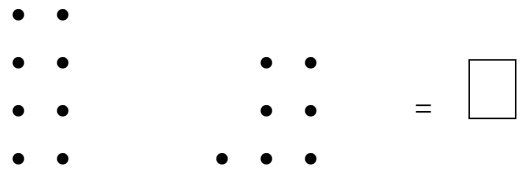
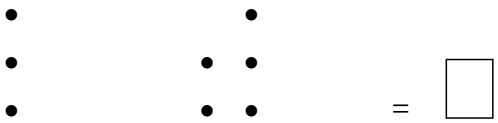
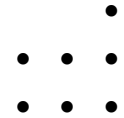
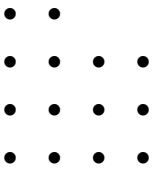
 = ☐	 = ☐
 = ☐	 = ☐



QA-A3

 = □	 = □

CQD-A

2 □ 20	7 × 50 □ 7 × 5	90 + 5 □ 91 + 5	10 □ 19 – 9
35 ÷ 5 □ 5	5 + 5 □ 55	31 □ 13	80 × 1 □ 8 × 10
10 □ 1 × 1	20 ÷ 4 □ 5	14 □ 14	40 □ 4
70 - 1 □ 70 - 8	80 □ 80 – 2	30 – 2 □ 30	57 □ 57
30 – 4 □ 30 – 2	50 □ 50 – 3	1 □ 10	72 ÷ 9 □ 8
80 + 7 □ 70 + 8	99 □ 9 × 10	40 ÷ 4 □ 30 ÷ 3	39 □ 93
10 + 4 □ 11 + 4	40 + 3 □ 30 + 4	15 □ 51	50 – 3 □ 51 – 3
60 – 7 □ 61 - 8	20 □ 20	75 □ 57	60 × 30 □ 60 × 3

$1 \times 3 \square 10$	$18 \div 6 \square 18 \div 3$	$32 \square 30 + 2$	$40 + 2 \square 20 + 4$

$74 \square 74$	$30 \square 30$	$22 \square 2 \times 10$	$40 \square 4$

EE-A1

38 x 5 2 20 200	Jim has 7 pens. His mom takes 2. How many pens now? 5 50 500	The team has 14 bats. They forgot 3 at the game. How many bats now? 1 11 110	Jacob read 69 books last year. Each book had 28 pages. How many pages did he read? 21 210 2,100
$2\overline{)58}$ 3 30 300	9 + 5 14 140 1,400	68 + 3 7 70 700	Ann has 9 hats. She gives 3 to Sue. How many hats now? 6 60 600
$285\overline{)597}$ 2 20 200	22 + 29 5 50 500	95 – 24 7 70 700	Ray has 28 toy cars. He puts 7 in each box. How many boxes? 4 40 400
Mike has 253 baseball cards. His dad gives him 109. How many cards does Mike have now? 3 35 350	$3\overline{)617}$ 2 20 200	13 – 8 5 50 500	543 x 9 450 4,500 45,000
58 stickers on the sheet. 2 boys must share. How many stickers each? 3 30 300	47 – 6 40 400 4,000	6 mats on the table. 4 cups on each mat. How many cups in all? 2 24 240	3 cats. Each cat has 13 toys. How many toys in all? 4 40 400

EE-A2

Dad gives 5 cookies to each of his 3 boys. How many cookies in all? 15 150 1,500	7 x 3 21 210 2,100	9 boys have 63 toys. How many toys for each boy? 7 70 700	$4\overline{)16}$ 4 40 400
There are 18 bowls. Mary puts 11 jelly beans in each bowl. How many jelly beans will she need? 2 20 200	There were 28 books in the box. Tina put in 19 more. Now how many books in the box? 5 50 500	$22\overline{)61}$ 3 30 300	4 + 6 1 10 100
22 x 89 180 1,800 18,000	Kim has 3 dogs. May has 2 dogs. How many dogs in all? 5 50 500	220 x 195 400 4,000 40,000	There are 57 french fries in the bag. Sally eats 21. How many fries are left? 40 400 4,000
702 – 198 5 50 500	Bill has 6 hats. He gets 2 more. How many hats now? 8 80 800	811 – 613 2 20 200	The class library has 391 books. Kids checked out 103 books. How many books now? 3 30 300
15 – 7 8 80 800	There are 192 pretzels in the bag and 18 kids in the class. How many pretzels for each kid? 10 100 1,000	19 cups on the table. Bob puts on 7 more. How many cups now? 2 26 260	1,500 pieces belong to 21 puzzles. About how many pieces in each puzzle? 7 70 700

Appendix B: Field of Problem Types for Missing Number and Complex Quantity Discrimination

Missing Number in Pattern: Field of Problems

Location of missing number will be the 4th position. 79 types of problem (below).

Count-by's on the multiple:
(Up to 20 for 1's - 4's, up to 50 for 5's – 9's, up to 100 for 10's; **57 problem types**)
1,2,3,4 ; 2,3,4,5 ; … 17,18,19,20 (17 problem types for the 1's…)
2,4,6,8 ; 4,6,8,10; …14,16,18,20 (7 problem types for the 2's…)
3,6,9,12 ; ……9,12,15,18 (3 problem types for 3's)
4,8,12,16 ; 8, 12, 16, 20 (2 types for 4's)
5,10,15,20 ; ……35,40,45,50 (7 types for 5's)
6,12,18,24 ; ……24,30,36,48 (5 types for 6's)
7,14,21,28 ; ……..,49 (4 types for 7's)
8,16,24,32 ; ……24,32,40,48 (3 types for 8's)
9,18,27,36 ; 18,27,36,45 (2 types for 9's)
10,… ; ……,100 (7 types for 10's)

Count-by's off the multiple:
(2's and 10's only; **22 problem types**)
2's:
1,3,5,7 ; 3,5,7,9 ; … 13,15,17,19 (7 problem types for 2's)
10's:
1,11,21,31 ; 11,21,31,41, ; 21,31,41,51 (3 types)
3,13,23,33……53 (" ")
5… …55
7… …57
9,19,29,39 ;… …29,39,49,59 (=15 problem types for 10's)

Complex Quantity Discrimination Problem Types

Included are examples of what is meant for the notation below where x=3 and y=7. Here, a comma indicates one digit after the other, not multiplied or added, so that x,y means 37. Where you see (xy), that *product* is included as the number shown on the probe.

Single Number / Single Number comparisons:

x & x,0 (e.g. 3 & 30)
x,y & y,x (37 & 73)
(xy) & (xy) (e.g. 21 & 21); or x,0 & x,0 (e.g. 30 & 30)

Single Number / Addition Operation:

x,0 & x0 + y (e.g. 30 & 30 + 7)
x,y & x + y (37 & 3+7)
x,y & x,0 + y (37 & 30 + 7)

Single Number / Subtraction Operation:

x,0 & x,0 – y (30 & 30 – 7)
x,0 & y,0 – x (30 & 70 – 3)
x,0 & x,y – y (30 & 37 – 7)

Single Number / Multiplication Operation:

x,y & x * 10 (37 & 3 * 10)
x,0 & x * y (30 & 3 * 7)
(xy) & x * y (21 & 3 * 7)

Single Number / Division Operation:

x & x,0 / [2 or 5] (e.g. 3 & 30 / 2)
x & (xy) / x (3 & 21 / 3)
x & (xy) / y (3 & 21 / 7)

Addition Operation / Addition Operation:

x,0 + y & (x,0+1) + y (30 + 7 & 31 + 7)
x,0 + y & y,0 + x
x,0 + y & y + x,0

Subtraction Operation / Subtraction Operation:

x,0 – y & (x,0 + 1) – y
x,0 – y & x,0 – z
x,0 – y & (x,0+1) – (y+1)

Multiplication Operation / Multiplication Operation:

x,0 * y,0 & x,0 * y
x * y,0 & x * y
x,0 * y & x * y,0

Division Operation / Division Operation:

(xy) / x & (xy) / y
(xy) / x & x,0 / x
x,0 / x & y,0 / y

The digits x and y (and z!) were generated randomly using 1 through 9.
The problem types were each filled in with digits enough times to make the number of items for the probe, then randomly selected without replacement to get them in an order.
Each probe included as many single number / single number comparisons as each of the other broad types.
Also note that the two sides of the box were switched to be the opposite position from those above in Problem Types about half of the time (random).

Appendix C

Teacher Rating Scale for Students' Math Proficiency

For each student below, please rate his or her ***general proficiency in math*** relative to other students in your class.

Try to spread student ratings across the ***full range of the scale***, not clustering students only in the middle or toward one end.

Thank you for your help!

Student Name	1 *(least proficient)*	2	3	4	5	6	7 *(most proficient)*
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7
	1	2	3	4	5	6	7

Appendix D: Directions and Examples Pages

Basic Facts

Cloze Math

Missing Number

Mixed Numeracy

Quantity Arrays

Complex Quantity Discrimination

Easy Estimation

Basic Math Facts: Directions

FORM 1

Say to the students:

"Turn to page ___ in your booklet, a yellow page. On this page are several kinds of arithmetic problems. Some are addition and subtraction and some are multiplication and division. Don't worry if you cannot do some problems. Just do as well as you can.

"In the first example, 10 minus 7 is…3. Write in 3.

"In the second example, 6 times 3 is…18. Write in 18.

"Now listen to directions.

"Look at each problem carefully before you answer it. When you begin, start on the first problem at the top and go **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, put an X on it and go on to the next one. Remember to look at each problem and put an X on it if you can't do it. Work **across** the page, from left to right, trying each problem in the row before you go down to the next row.

"Try some of these problems on the bottom of the yellow page just for practice. (20 seconds only.) OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. You will have one minute to work. Stay on the pink page only."

"Ready? Begin." Start stopwatch as you say BEGIN.

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another page of arithmetic problems. Remember to start on the first problem at the top and work **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the green page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last page of arithmetic problems. Remember to start on the first problem and work **across** each row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the white page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page."

Examples:

A) 10
- 7

Answer: *10*
- 7
3

B) 6
× 3

Answer: *6*
×3
18

Try these:

4 + 9	11 - 8	0 × 3	$1\overline{)4}$	12 - 5	$4\overline{)28}$	6 × 4	8 - 3	7 + 6	1 + 8

8 - 2	4 + 0	9 × 0	1 × 3	10 - 4	2 × 7	$8\overline{)32}$	5 + 5	3 + 9	6 × 6

Cloze Math Facts: Directions

FORM 1

Say to the students:

"Turn to page ___ in your booklet, a yellow page. On this page are several kinds of number sentences. Some are addition or subtraction and some are multiplication or division. Each one has a number missing within the number sentence, and in its place is an empty box. Your job is to write the number in the box that makes the number sentence true.

"Look at the example problems. The first one says, 'six plus **something** equals seven.' The number that goes in the box to make this true is…one. Now look at the second example. It says, '**something** times two equals six.' The number that goes in the box to make this true is…three. Don't worry if you cannot do some problems. Just do as well as you can.

"Now listen to directions.

"When you begin, start on the first problem at the top and go **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, put an X on it and go on to the next one. Remember to look at each problem and put an X on it if you can't do it. Work **across** the page, from left to right, trying each problem in the row before you go down to the next row.

"Try some of these problems on the bottom of the yellow page just for practice. (20 seconds only.) OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. You will have one minute to work. Stay on the pink page only."

"Ready? Begin." Start stopwatch as you say BEGIN.

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another page of number sentences with some numbers missing. Remember to start on the first problem at the top and work **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the green page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last page of number sentences. Remember to start on the first problem and work **across** each row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the white page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page."

Examples:

A) 6 + □ = 7

Answer: *6 + □ = 7*

B) □ x 2 = 6

Answer: *□ x 2 = 6*

Try these:

11 – 3 = □	□ + 2 = 8	3 × □ = 9	15 ÷ 5 = □
□ × 9 = 27	9 + □ = 10	5 + 3 = □	□ ÷ 4 = 4
10 ÷ □ = 2	4 × 3 = □	□ – 9 = 0	40 ÷ □ = 5

Missing Number in Pattern: Directions

FORM 1

Say to the students:

"Turn to page ___ in your booklet, a yellow page. On this page are different kinds of number patterns. The last number in each pattern is missing. Some patterns go up by ones or twos. Other patterns go up by more, like sevens or even tens. Look at each pattern carefully before you fill in the blank. In the blank, write the number that comes next in the pattern.

"Look at the example patterns. In the first one, 2, 4, 6, ___ , the pattern is going up by twos. The number that comes next in the pattern is…8. Write in 8.

In the second example, 5, 15, 25, ___, the pattern is going up by tens. The number that comes next in this pattern is…35. Write in 35.

In the third example, 10, 9, 8, ___, the pattern is going down by ones. The number that comes next in this pattern is…7. Write in 7.

"Now listen to directions.

"When you begin, start on the first pattern at the top and go **across** that row, then begin the next row. Try every pattern. If you come to one you cannot answer, put an X on it and go on to the next one. Remember to look at each pattern and put an X on it if you can't do it. Work **across** the page, from left to right, trying each pattern in the row before you go down to the next row.

"Try some of these problems on the bottom of the yellow page just for practice. *(20 seconds only.)* OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. You will have one minute to work. Stay on the pink page only."

"Ready? Begin." Start stopwatch as you say BEGIN.

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another page of patterns. Remember to start on the first pattern at the top and work **across** each row. Try every pattern. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each pattern.

"Stay on the green page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last page of patterns. Remember to start on the first problem and work **across** each row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the white page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page."

Examples:

A) 2, 4, 6, ___

Answer: *2, 4, 6,* ___

B) 5, 15, 25, ___

Answer: 5, *15, 25,* ___

C) 8, 9, 10, ___

Answer: *8, 9, 10,* ___

Try these:

6, 8, 10, ___	13, 23, 33, ___	6, 16, 26, ___	15, 20, 25, ___
50, 60, 70, ___	2, 4, 6, ___	8, 9, 10, ___	60, 70, 80, ___
10, 15, 20, ___	8, 12, 16, ___	3, 5, 7, ___	11, 21, 31, ___

Quantity Arrays: Directions

FORM 1

Say to the students:

"Turn to page ___ in your booklet, a yellow page. On this page are groups of dots. Your job is to figure out how many dots there are in each problem. For some problems, you might recognize how many dots there are very quickly. For others, it might take more time to figure out how many dots are there. For every problem, write in the empty box the number of dots that you see.

"Sometimes the dots are arranged in two groups, and your job is to figure out how many dots there are *in all* for that problem. When you have figured out how many dots there are **altogether**, write that number in the empty box.

Look at the first example. How many dots are there? There are 6. Write 6 in the empty box.

Look at the second example. How many dots are there in all? There are five and five, so the total number of dots there is…10. Write 10 in the empty box.

"Now listen to directions.

"Look at each problem carefully before you answer it. Start on the first problem at the top and go **across** that row, then begin the next row. Write an answer for every problem. Do not skip. Work **across** the page, from left to right, doing each problem in the row before you go on to the next row.

"Try some of these problems on the bottom of the yellow page just for practice. *(20 seconds only.)* OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. Keep pencils down. There are **three** pink pages of dots and you will have one minute to work on them. When you finish a page, go on to the next pink page. If you go past the third pink page, the next page will tell you to stop.

"Stay on the pink pages only. Ready? Begin." Start stopwatch as you say BEGIN.

(After one minute, say): "Stop. Thank you, put your pencils down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another set of dots problems. Remember to start on the first problem at the top and work **across** each row. If you finish a page, go on to the next green page.

"Stay in the green pages only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last set of dots problems. Remember to start on the first problem at the top and work **across** each row. If you finish a page, go on to the next white page.

"Stay in the white pages only. Ready? Begin."

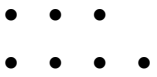
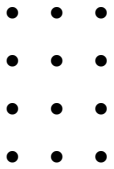
(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page. All pencils should be down."

Examples:

A)  = ☐

B)  = ☐

Try these:

 = ☐	 = ☐
 = ☐	 = ☐
 = ☐	 = ☐

Mixed Numeracy: Directions

FORM 1

Say to the students:

"Turn to page ___ in your booklet, a yellow page. On this page are three different kinds of problems- number sentences, number patterns, and arithmetic problems. You've done them all before, either today or the last time I was here.

"Look at the example problems. The first one is a number sentence. It says, '6 plus **something** equals 7.' The number that goes in the box to make this true is…one. Remember, some number sentences are addition or subtraction and some are multiplication or division. Each one has a number missing within the number sentence, and in its place is an empty box. Your job is to write the number in the box that makes the number sentence true.

"Now look at the second example. It is a number pattern. In 60, 70, 80, ___ , the pattern is going up by tens. The number that comes next in the pattern is…90. Write in 90. Remember, for the number patterns, the last number in each pattern is missing. Some patterns go up by ones or twos. Other patterns go up by more, like sevens or even tens. Look at each pattern carefully before you fill in the blank. In the blank, write the number that comes next in the pattern.

"Now look at the third example. It is an arithmetic problem. 4 plus 9 is…13. Write in 13. Remember, some arithmetic problems are addition and subtraction and some are multiplication and division.

"Don't worry if you cannot do some problems. Just do as well as you can.

"Now listen to directions.

"When you begin, start on the first problem at the top and go **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, put an X on it and go on to the next one. Remember to look at each problem and put an X on it if you can't do it. Work **across** the page, from left to right, trying each problem in the row before you go down to the next row.

"Try some of these problems on the bottom of the yellow page just for practice. (20 seconds only.) OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. You will have one minute to work. Stay on the pink page only."

"Ready? Begin." Start stopwatch as you say BEGIN.

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another page of number sentences, number patterns, and arithmetic problems. Remember to start on the first problem at the top and work **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the green page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last page of number sentences, number patterns, and arithmetic problems. Remember to start on the first problem and work **across** each row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the white page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page."

Examples:

A) $6 + \square = 7$

Answer: *$6 + \square = 7$*

B) 60, 70, 80, ___

Answer: *60, 70, 80,___*

C)
$$\begin{array}{r} 4 \\ +\ 9 \\ \hline \end{array}$$

Answer:
$$\begin{array}{r} 4 \\ +9 \\ \hline 13 \end{array}$$

Try these:

2, 4, 6, ___	$\square + 2 = 8$	$\begin{array}{r} 5 \\ +\ 5 \\ \hline \end{array}$	3, 5, 7, ___
$\square \times 9 = 27$	$4\overline{)28}$	10, 9, 8, ___	$\begin{array}{r} 11 \\ -\ 8 \\ \hline \end{array}$
$10 \div \square = 2$	6, 16, 26, ___	$\square - 9 = 0$	$\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$

Easy Estimation: Directions

FORM 1

(*Say to the students*:)

"Turn to page ___ in your booklet, a yellow page. On this page are estimation problems. For each problem, you will see three answer choices. Choose the number that is correct or comes closest to the correct answer.

"Look at the first example. 69 is about like 70, and 8 is about like 10. So 69 – 8 is about…60. 60 is circled as the best answer.

"Look at the second example. It says, *There are 19 men. Each man has 11 hats. How many hats in all?* Now, 19 men is about 20, and 11 hats each is about 10 each. So this is about like 20 men with 10 hats each. About how many hats in all? In all, about…200. Circle 200 as the best choice.

"Now listen to directions.

"Work carefully and do the best you can. When you begin, start at the top left. Work from left to right. Some problems will be easy for you; others will be harder. When you come to a problem that's hard for you, skip it, and come back to it later. Go through the entire test doing the easy problems. Then go back and try the harder ones.

"Try some of these problems on the bottom of the yellow page just for practice. *(20 seconds only.)* OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. This part has two pink pages and a total of forty math problems. If you go past the second pink page, the next page says to STOP. You can go back to problems in the two pink pages that you skipped, but only in this 2 minutes.

"Stay on the pink pages only. You will have two minutes. Ready? Begin." (*Start stopwatch as you say BEGIN*.)

(*After 2 minutes, say*:) "Stop. Thank you, put your pencil down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another set of estimation problems. Remember to start on the first problem at the top and work **across** each row. If you come to a problem that is harder for you, skip it and come back to it later.

"Stay on the two green pages only. Ready? Begin."

(After 2 minutes, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last set of estimation problems. Remember to work **across** each row. If you come to a problem that is harder for you, skip it and come back to it later.

"Stay on the two white pages only. Ready? Begin."

(After 2 minutes, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page. All pencils should be down."

Need Easy Estimation examples page here

Complex Quantity Discrimination: Directions

FORM 1

Say to the students:

"Turn to page ___ in your booklet, a yellow page. On this page are number comparisons. One on side of an empty box is an amount, and on the other side of the box is another amount. Your job is to decide which one is bigger. Sometimes the amount is just a number. Other times, you might have to figure out the amount by doing addition, subtraction, multiplication, or division. As you think about the amounts, your job is to decide which amount is larger.

"If both of the amounts are equal, then you should put an equal sign in the box between them. If one of the amounts is greater than the other, then you should put a "greater than" or "less than" sign in the box to show which is greater.

"Look at the first example on this page. One amount is 20 + 4, and the other amount is 20. Since 24 is greater than 20, you should put a "greater than" sign, as is shown. Remember, the sign looks like an alligator mouth that is opened towards the greater amount.

"Now look at the second example. One amount is 5, and the other amount is 5 x 10. Since 5 is less than 50, you should put a "less than" sign, like is shown. Remember, the sign opens its mouth towards the bigger amount.

Now look at the third example. One amount is 8, and the other amount is 10 – 2. Since 8 is equal to 8, you should put an equal sign, like is shown. For every problem, you put either an equal sign, a "greater than" sign, or a "less than" sign in the box."

"Listen to these directions. Look at each problem carefully before you answer it. When you begin, start on the first problem at the top and go **across** that row, then begin the next row. Try every problem. If you come to one you cannot answer, put an X on it and go on to the next one. Remember to look at each problem and put an X on it if you can't do it. Work **across** the page, from left to right, doing each problem in the row before you go down to the next row.

"Try some of these problems on the bottom of the yellow page just for practice. (20 seconds only.) OK, now are there any questions before we turn the page?"

"Put pencils down. Turn now to the next page, which is pink. You will have one minute to work. Stay on the pink page only."

"Ready? Begin." Start stopwatch as you say BEGIN.

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page, which is a green page. All pencils should be down."

FORM 2

Say to the students:

"Now you will try another page of number comparisons. You should put an equal sign if the amounts are equal. If they are not equal, remember that the greater than and less than signs look like an alligator mouth. The alligator opens its mouth towards the ***larger*** amount.

"Remember to work **across** each row. Try every problem. If you come to one you cannot answer, you can put an X on it and go on to the next one, but try each problem.

"Stay on the green page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a white page. All pencils should be down."

FORM 3

Say to the students:

"Now you will try one last page of number comparisons. Put an equal sign, a less than sign, or a greater than sign in the empty box. Remember that the alligator opens its mouth towards the ***larger*** amount.

"Work **across** each row. Try every problem, and put an X if you cannot answer it.

"Stay on the white page only. Ready? Begin."

(After one minute, say): "Stop. Thank you, put your pencil down. Turn to the next page in your booklet, which is a yellow page. All pencils should be down."

Examples:

A) 20 + 4 □ 20

Answer: *20 + 4 □ 20*

B) 5 □ 5 x 10

Answer: *5 □ 5 x 10*

C) 8 □ 10 – 2

Answer: *8 □ 10 – 2*

Try these:

10 - 1 □ 10 - 6	30 □ 30 + 2	70 × 2 □ 2 × 70	27 □ 72
15 ÷ 5 □ 5	4 + 4 □ 44	31 □ 31	60 × 1 □ 6 ×10
70 – 7 □ 70 – 1	20 □ 20 – 3	3 □ 30	40 ÷ 5 □ 8
1 × 4 □ 10	20 ÷ 2 □ 20 ÷ 10	46 □ 40 + 6	20 + 4 □ 40 + 2